Markscheme

November 2020

Mathematical studies

Standard level

Paper 2

28 pages

Paper 2 Markscheme
Instructions to Examiners

Notes: **If in doubt about these instructions or any other marking issues, contact your team leader for clarification.**

1   Abbreviations

M   Marks awarded for **Method**

A   Marks awarded for an **Answer** or for **Accuracy**

R   Marks awarded for clear **Reasoning**

G   Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

AG   **Answer Given** in the question and consequently, marks not awarded.

ft   Marks that can be awarded as **follow through** from previous results in the question.

2   Method of Marking

(a)   All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.

(b)   If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the *A0* annotation, otherwise full annotations must be shown.

(c)   Working crossed out by the candidate should not be awarded any marks.

(d)   Where candidates have written two solutions to a question, only the first solution should be marked.

(e)   If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final *(A1)* *(ignore the further working)*
2.	$(x-6)(x+1)$	$x=6$ and -1	Do **not** award the final *(A1)*

Example: Calculate the gradient of the line passing through the points $(5,3)$ and $(0,9)$.

Markscheme	Candidates' Scripts	Marking
$\dfrac{9-3}{0-5}$ *(M1)* *Award (M1) for correct substitution in gradient formula* $=-\dfrac{6}{5}$ *(A1)*	(i) $\dfrac{9-3}{0-5}=-\dfrac{6}{5}$ Gradient is $=-\dfrac{6}{5}$ *(There is clear understanding of the gradient.)* $y=-\dfrac{6}{5}x+9$ (ii) $\dfrac{9-3}{0-5}=-\dfrac{6}{5}$ $y=-\dfrac{6}{5}x+9$ *(There is confusion about what is required.)*	*(M1)* *(A1)* *(M1)* *(A0)*

3   Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

(a)   Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.

(b)   If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.

(c)   If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.

(d)   To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.

(e)   The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**

(f)   Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
(a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ **(M1)(A1)**	(a) $\dfrac{\sin A}{4} = \dfrac{\sin 30}{3}$	**(M1)(A0)**
Award **(M1)** *for substitution in sine rule formula,* **(A1)** *for correct substitutions.*		*(use of sine rule but with wrong values)*
$A = 22.0°\ (22.0243\ldots)$ **(A1)(G2)**	$A = 41.8°$	**(A0)**
		*(**Note:** the 2nd **(A1)** here was not marked **(ft)** and cannot be awarded because there was an earlier error in the **same** question part.)*
(b) $x = 7\tan(22.0243\ldots°)$ **(M1)** $= 2.83\,(2.83163\ldots)$ **(A1)(ft)**	(b) case (i) $x = 7\tan 41.8°$ $= 6.26$	**(M1)** **(A1)(ft)**
	but case (ii) 6.26	**(G0)** *since no working shown*

4   Using the Markscheme

(a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

(b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.

(c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

Example: Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ **(M1)(A1)** *Award **(M1)** for substitution in sine rule formula, **(A1)** for correct substitutions.* $A = 22.0°$ $(22.0243\ldots)$ **(A1)(G2)**	(i) $\dfrac{\sin A}{3} = \dfrac{\sin 30}{4}$ $A = 22.0°$ (ii) $A = 22.0°$ **Note: G** marks are used only if no working has been shown and the answer is correct.	**(M1)(A1)** **(A1)** **(G2)**

(d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(e) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\dfrac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value $\left(\text{for example } \dfrac{2}{3} \text{ if applicable}\right)$;
the full calculator display in the form $2.83163\ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(f)   As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; $1'7$; $1 \cdot 7$; $1,7$.

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or $.49$.

Different descriptions of an interval: $3 < x < 5$; $(3, 5)$; $\,]\,3, 5\,[$.

Different forms of notation for set properties (*e.g.* complement): A'; $\overline{A}$; A^c; $U - A$; $(A\,$; $U \setminus A$.

Different forms of logic notation: $\neg p$; p'; $\tilde{p}$; $\overline{p}$; $\sim p$; $-p$.

$$p \Rightarrow q\,;\ p \rightarrow q\,;\ q \Leftarrow p\,.$$

Significance level may be written as α .

(g)   Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5   Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1.   If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(A1)** and ignore subsequent rounding.

2.   If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.
Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3.   If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given …					
	Exact or to 4 or more sf (and would **round to the correct 3 sf**)	**Correct to 3 sf**	**Incorrect to 3 sf**	Correct to 2 sf [3]	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen[1]	Award the final **(A1)** irrespective of correct or incorrect rounding					
Unrounded answer not seen[2]	**(A1)**	**(A1)**	**(A0)**	**(A1)**	**(A0)**	**(A0)**
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen.[3]			

Examples:

Markscheme	Candidates' Scripts		Marking
9.43 (9.43398…) **(A1)**	(i)	9.43398… is seen followed by 9; 9.4; 9.43; 9.434 *etc.* (correctly rounded)	**(A1)**
	(ii)	9.43398… is seen followed by 9.433; 9.44 *etc.* (incorrectly rounded)	**(A1)**
	(iii)	9.4	**(A1)**
	(iv)	9	**(A0)** *(correct to 1 sf)*
	(v)	9.3	**(A0)** *(incorrectly rounded to 2 sf)*
	(vi)	9.44	**(A0)** *(incorrectly rounded to 3 sf)*

Markscheme	Candidates' Scripts		Marking
7.44 (7.43798…) **(A1)**	(i)	7.43798… is seen followed by 7; 7.4; 7.44; 7.438 *etc.* (correctly rounded)	**(A1)**
	(ii)	7.43798… is seen followed by 7.437; 7.43 *etc.* (incorrectly rounded)	**(A1)**
	(iii)	7.4	**(A1)**
	(iv)	7	**(A0)** *(correct to 1 sf)*
	(v)	7.5	**(A0)** *(incorrectly rounded to 2 sf)*
	(vi)	7.43	**(A0)** *(incorrectly rounded to 3 sf)*

Example: ABC is a right angled triangle with angle $ABC = 90°$, $AC = 32\text{ cm}$ and $AB = 30\text{ cm}$. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $\quad BC = \sqrt{32^2 - 30^2}$ **(M1)** *Award* **(M1)** *for correct substitution in Pythagoras' formula* $\quad = 11.1\left(\sqrt{124}, 11.1355...\right)(\text{cm})$ **(A1)** (b) $\quad \text{Area} = \dfrac{1}{2} \times 30 \times 11.1355...$ **(M1)** *Award* **(M1)** *for correct substitution in area of triangle formula* $\quad = 167(167.032...)(\text{cm}^2)$ **(A1)(ft)**	(a) $\quad BC = \sqrt{32^2 - 30^2}$ **(M1)** $\quad 11\,(\text{cm})$ **(A1)** *(2 sf answer only seen, but correct)* (b) case (i) $\quad \text{Area} = \dfrac{1}{2} \times 30 \times 11$ **(M1)** *(working shown)* $\quad = 165\,(\text{cm}^2)$ **(A1)(ft)** case (ii) $\quad = 165\,(\text{cm}^2)$ **(M0)(A0)(ft)** *(No working shown, the answer 11 is treated as a ft, so no marks awarded here)*

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared $7.68\,(7.67543\ldots)$ **(A2)**	(a) $\quad 7.7$ (b) $\quad 7.67$ (c) $\quad 7.6$ (d) $\quad 8$ (e) $\quad 7$ (e) $\quad 7.66$	**(G2)** **(G1)** **(G1)** **(G0)** **(G0)** **(G0)**

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ **(A2)** $(y = 0.887686…x + 13.4895…)$ If an answer is not in the form of an equation award at most **(A1)(A0)**.	(a) $y = 0.89x + 13$	**(G2)** *(both accepted)*
	(b) $y = 0.88x + 13$	**(G1)** *(one rounding error)*
	(c) $y = 0.88x + 14$	**(G1)** *(rounding error repeated)*
	(d) (i) $y = 0.9x + 13$	
	(ii) $y = 0.8x + 13$	**(G1)** *(1 sf not accepted)*
	(e) $0.88x + 13$	**(G0)** *(one rounding error and not an equation)*

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ **(A1)(A1)** $(2.06020…, 4.49253…)$	(a) $(2.1, 4.5)$	**(A1)(A1)** *(both accepted)*
	(b) $(2.0, 4.4)$	**(A1)** *(same rounding error twice)*
	(c) $(2.06, 4.4)$	**(A1)** *(one rounding error)*
	(d) $(2, 4.4)$	**(A0)** *(1sf not accepted, one rounding error)*

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\dfrac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (*eg,* $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg,* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

6 **Level of accuracy in finance questions**

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
$231.62 (231.6189) *(A1)*	(i) 231.6	*(A0)*
	(ii) 232	*(A0)* *(Correct rounding to incorrect level)*
	(iii) 231.61	*(A0)*
	(iv) 232.00	*(A0)* *(Parts (iii) and (iv) are both incorrect rounding to correct level)*

7 **Units in answers**

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded *(A1)* under the accuracy rules given in Section 5.

Example:

Markscheme	Candidates' Scripts	Marking
(a) $37000 \, \text{m}^2$ *(A1)*	(a) $36000 \, \text{m}^2$	*(A0)* *(Incorrect answer so units not considered)*
(b) $3200 \, \text{m}^3$ *(A1)*	(b) $3200 \, \text{m}^2$	*(A0)* *(Incorrect units)*

If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

8 **Graphic Display Calculators**

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a)
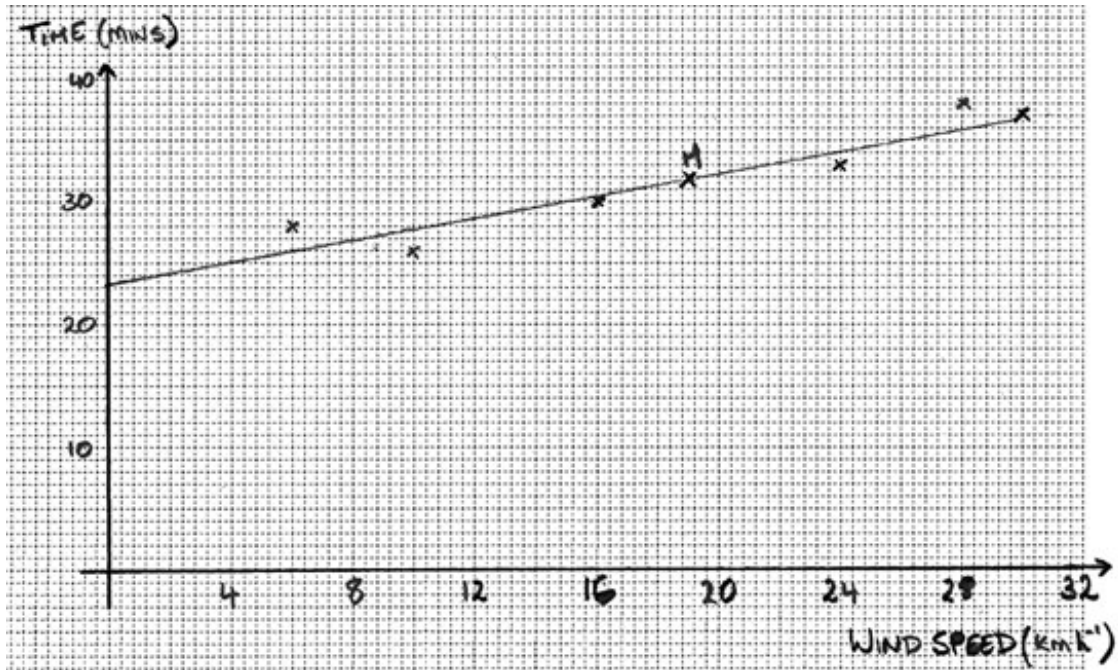
Note:	Award *(A1)* for correct scales and labels.
	Award *(A3)* for all six points correctly plotted.
	Award *(A2)* for four or five points correctly plotted.
	Award *(A1)* for two or three points correctly plotted.
	Award at most *(A0)(A3)* if axes reversed.
	If graph paper is not used, award at most *(A1)(A0)(A0)(A0)*.

(A4)

[4 marks]

(b) (i)   19 $(km\ h^{-1})$ *(A1)*

(ii)   32 (minutes)   *(A1)*

[2 marks]

(c)   point in correct position, labelled M   *(A1)(ft)(A1)*

Note:	Award *(A1)(ft)* for point plotted in correct position, *(A1)* for point labelled M
	Follow through from their part (b).

[2 marks]

continued…

Question 1 continued

(d) (i) $(r =)$ 0.944 (0.943733...)   **(G2)**

> **Note:**   Award **(G1)** for 0.943 (incorrect rounding).

(ii)   (very) strong positive correlation   **(A1)(ft)(A1)(ft)**

> **Note:**   Award **(A1)(ft)** for (very) *strong*. Award **(A1)(ft)** for *positive*. Follow though from their part (d)(i). If there is no answer to part (d)(i), award at most **(A0)(A1)** for a correct direction.

[4 marks]

(e) (i) $y = 0.465x + 23.2$ ($y = 0.465020\ldots x + 23.1646\ldots$)   **(A1)(A1)(G2)**

> **Note:**   Award **(A1)** for $0.465x$. Award **(A1)** for 23.2. If the answer is not an equation, award at most **(A1)(A0)**.

(ii)   regression line through their M   **(A1)(ft)**
        regression line through their (0, 23.2)   **(A1)(ft)**

> **Note:**   Award a maximum of **(A1)(A0)** if the line is not straight/ruler not used. Award **(A0)(A0)** if the points are connected.
>
> Follow through from their point M in part (b) and their y-intercept in part (e)(i).
>
> If M is not plotted or labelled, then follow through from part (b).

Question 1(e) continued

(iii) $(y =)\ 0.465020\ldots(27) + 23.1646\ldots$ **(M1)**

> **Note:** Award **(M1)** for correct substitution into their regression equation.

35.7 (minutes) (35.7201...)   **(A1)(ft)(G2)**

> **Note:** Follow through from their equation in part (e)(i).

OR

an attempt to use their regression line to find the y value at $x = 27$ **(M1)**

> **Note:** Award **(M1)** for an indication of using their regression line. This must be illustrated by vertical **and** horizontal lines or marks at the correct place(s) on their scatter diagram.

35.7 (minutes)   **(A1)(ft)**

> **Note:** Follow through from part (e)(ii).

                    [6 marks]

(f)   wind speed **does not cause a change** in the time to charge (the robot)   **(A1)**

> **Note:** Award **(A1)** for a statement that communicates the meaning of a non-causal relationship between the two variables.

                    [1 mark]
                    Total [19 marks]

2.   (a)   (i) $74 - (32 + 12 + 10 + 9 + 5)$ **OR** $74 - 68$ *(M1)*

> **Note:** Award *(M1)* for setting up a correct expression.

$(a =)\ 6$ *(A1)(G2)*

(ii) $100 - (74 + 18)$ *(M1)*

OR

$100 - 92$ *(M1)*

OR

$100 - (32 + 9 + 5 + 12 + 10 + 18 + 6)$ *(M1)*

> **Note:**   Award *(M1)* for setting up a correct expression. Follow through from part (a)(i) but only for $a \geq 0$.

$(b =)\ 8$ *(A1)(ft)(G2)*

> **Note:**   Follow through from part(a)(i). The value of b must be greater or equal to zero for the *(A1)(ft)* to be awarded.

[4 marks]

(b) $9 + 5 + 12 + 10$ *(M1)*

> **Note:**   Award *(M1)* for adding 9, 5, 12, and 10.

36   *(A1)(G2)*

[2 marks]

continued…

Question 2 continued

(c)   14   *(A1)*

                                                                                              [1 mark]

(d)   (i) $\dfrac{58}{100}$ $\left(\dfrac{29}{50}, 0.58, 58\%\right)$ *(A1)(A1)(G2)*

> **Note:**   Award *(A1)* for correct numerator. Award *(A1)* for the correct
>   denominator. Award *(A0)* for 58 only.

      (ii) $\dfrac{45}{100}$ $\left(\dfrac{9}{20}, 0.45, 45\%\right)$ *(A1)(ft)*

> **Note:**   Follow through from their denominator from part (d)(i).

                                                                                              [3 marks]

(e)   they are not independent   *(A1)(ft)*

$\dfrac{58}{100} \times \dfrac{45}{100} \neq \dfrac{17}{100}$ **OR** $0.261 \neq 0.17$ *(R1)*

> **Note:**   Comparison of numerical values must be seen for *(R1)* to be awarded.
>   Do not award *(A1)(R0)*. Follow through from parts (d)(i) and (d)(ii).

                                                                                              [2 marks]

                                                                                       Total [12 marks]

3.   *Units are required in part (c).*

(a) $\dfrac{BD}{\sin 51.5°} = \dfrac{8}{\sin 52.5°}$ *(M1)(A1)*

> **Note:**   Award *(M1)* for substituted sine rule, *(A1)* for correct substitution.

   $(BD =)$ 7.89 (cm) (7.89164...)   *(A1)(G2)*

> **Note:**   If radians are used the answer is 9.58723… award at most *(M1)(A1)(A0)*.

                                                                    [3 marks]

(b) $\cos \text{EDC} = \dfrac{9^2 + 3.94582...^2 - 7^2}{2 \times 9 \times 3.94582...}$ *(A1)(ft)(M1)(A1)(ft)*

> **Note:**   Award *(A1)* for 3.94582… or $\dfrac{7.89164...}{2}$ seen, *(M1)* for substituted cosine rule, *(A1)(ft)* for correct substitutions.

   $(EDC =)$ 47.9515...°   *(A1)*
   48.0° (3 sig figures)   *(AG)*

> **Note:**   Both an unrounded answer that rounds to the given answer
>   and the rounded value must be seen for the final *(A1)* to be awarded.
>   Award at most *(A1)(ft)(M1)(A1)(ft)(A0)* if the known angle 48.0° is used
>   to validate the result. Follow through from their BD in part (a).

                                                                    [4 marks]

(c)   (area =) $\dfrac{1}{2} \times 7.89164... \times 9 \times \sin 48.0°$ *(M1)(A1)(ft)*

> **Note:**   Award *(M1)* for substituted area formula. Award *(A1)* for correct substitution.

   (area =) 26.4 cm² (26.3908...)   *(A1)(ft)(G3)*

> **Note:**   Follow through from part (a).

                                                                    [3 marks]

continued…

Question 3 continued

(d) $AE^2 = 8^2 + (3.94582...)^2 - 2 \times 8 \times 3.94582...\cos(76°)$ **(A1)(M1)(A1)(ft)**

> **Note:**   Award **(A1)** for $76°$ seen. Award **(M1)** for substituted cosine rule to find AE, **(A1)(ft)** for correct substitutions.

   $(AE =)$ 8.02 (cm) (8.01849...)   **(A1)(ft)(G3)**

> **Note:** Follow through from part (a).

   OR
   $AE^2 = 9.78424...^2 + (3.94582...)^2 - 2 \times 9.78424... \times 3.94582...\cos(52.5°)$
                                                              (A1)(M1)(A1)(ft)

> **Note:**   Award **(A1)** for AD (9.78424...) or $76°$ seen. Award **(M1)** for substituted cosine rule to find AE (do not award **(M1)** for cosine or sine rule to find AD), **(A1)(ft)** for correct substitutions.

   $(AE =)$ 8.02 (cm) (8.01849...)   **(A1)(ft)(G3)**

> **Note:** Follow through from part (a).

   $8.02 > 7$.   **(A1)(ft)**
   point A is outside the circle.   **(AG)**

> **Note:** Award **(A1)** for a numerical comparison of AE and CE. Follow through for the final **(A1)(ft)** within the part for their 8.02. The final **(A1)(ft)** is contingent on a valid method to find the value of AE.
> Do not award the final **(A1)(ft)** if the **(AG)** line is not stated.
> Do not award the final **(A1)(ft)** if their point A is inside the circle.

[5 marks]

[Total 15 marks]

4. (a) $h^2 + r^2 = 50^2$ (or equivalent)   **(A1)**

> **Note:** Accept equivalent expressions such as $r = \sqrt{2500 - h^2}$ or $h = \sqrt{2500 - r^2}$.
>
> Award **(A0)** for a final answer of $\pm\sqrt{2500 - h^2}$ or $\pm\sqrt{2500 - r^2}$, or any further incorrect working.

[1 mark]

(b) $\dfrac{1}{3} \times \pi \times \left(2500 - h^2\right) \times h$ **OR** $\dfrac{1}{3} \times \pi \times \left(\sqrt{2500 - h^2}\right)^2 \times h$ **(M1)**

> **Note:** Award **(M1)** for correct substitution in the volume of cone formula.

$V = \dfrac{2500\pi h}{3} - \dfrac{\pi h^3}{3}$ **(AG)**

> **Note:** The final line must be seen, with no incorrect working, for the **(M1)** to be awarded.

[1 mark]

(c) $\left(\dfrac{\mathrm{d}V}{\mathrm{d}h} =\right) \dfrac{2500\pi}{3} - \pi h^2$ **(A1)(A1)**

> **Note:** Award **(A1)** for $\dfrac{2500\pi}{3}$, **(A1)** for $-\pi h^2$. Award at most **(A1)(A0)** if extra terms are seen. Award **(A0)** for the term $-\dfrac{3\pi h^2}{3}$.

[2 marks]

(d) $0 = \dfrac{2500\pi}{3} - \pi h^2$ **(M1)**

> **Note:** Award **(M1)** for equating **their** derivative to zero. Follow through from part (c).

continued…

Question 4(d) continued

OR

sketch of $\dfrac{dV}{dh}$ *(M1)*

> **Note:** Award *(M1)* for a labelled sketch of $\dfrac{dV}{dh}$ with the curve/axes correctly labelled *or* the x-intercept explicitly indicated.

$(h =)$ 28.9 (cm) $\left(\sqrt{\dfrac{2500}{3}},\ \dfrac{50}{\sqrt{3}},\ \dfrac{50\sqrt{3}}{3},\ 28.8675... \right)$ *(A1)(ft)*

> **Note:** An unsupported 28.9 cm is awarded no marks. Graphing the **function** $V(h)$ is not an acceptable method and *(M0)(A0)* should be awarded. Follow through from part (c). Given the restraints of the question, $h \geq 50$ is not possible.

[2 marks]

(e) $(V =) \dfrac{2500 \times \pi \times 28.8675...}{3} - \dfrac{\pi (28.8675...)^3}{3}$ *(M1)*

OR

$\dfrac{1}{3} \pi (40.828...)^2 \times 28.8675...$ *(M1)*

> **Note:** Award *(M1)* for substituting their 28.8675... in the volume formula.

$(V =)$ 50400 (cm^3) (50383.3...)   *(A1)(ft)(G2)*

> **Note:** Follow through from part (d).

[2 marks]

(f) $(S =) \pi \times \sqrt{2500 - (28.8675...)^2} \times 50$ *(A1)(ft)(M1)*

> **Note:** Award *(A1)* for their correct radius seen $\left(40.8248...,\ \sqrt{2500 - (28.8675...)^2} \right)$.
>
> Award *(M1)* for correctly substituted curved surface area formula for a cone.

$(S =)$ 6410 (cm^2) (6412.74...)   *(A1)(ft)(G2)*

> **Note:** Follow through from parts (a) and (d).

[3 marks]
Total [11 marks]

5.   (a) $\sin 60° = \dfrac{h}{40}$ **OR** $\tan 60° = \dfrac{h}{20}$ *(M1)*

> **Note:**   Award *(M1)* for correct substitutions in trig ratio.

   OR
   $20^2 + h^2 = 40^2 \qquad \left(\sqrt{40^2 - 20^2}\right)$ *(M1)*

> **Note:**   Award *(M1)* for correct substitutions in Pythagoras' theorem.

   $(h =)$ 34.6 (m) $(\sqrt{1200},\ 20\sqrt{3},\ 34.6410...)$ *(A1)(G2)*

                                                                        [2 marks]

(b) $\dfrac{1}{2}(70+110)(34.6410...) \times 200$ *(M1)(M1)*

> **Note:**   Award *(M1)* for their correctly substituted area of trapezium formula, provided all substitutions are positive. Award *(M1)* for multiplying by 200. Follow through from part (a).

   OR
   $\left(2 \times \dfrac{1}{2} \times 20 \times 34.6410... + 70 \times 34.6410...\right) \times 200$ *(M1)(M1)*

> **Note:**   Award *(M1)* for the addition of correct areas for two triangles and one rectangle. Award *(M1)* for multiplying by 200. Follow through from part (a).

   OR
   $70 \times 34.6410... \times 200 + 2 \times \dfrac{1}{2} \times 34.6410... \times 20 \times 200$ *(M1)(M1)*

> **Note:**   Award *(M1)* for their correct substitution in volume of cuboid formula. Award *(M1)* for correctly substituted volume of triangular prism(s). Follow through from part (a).

   623538...   *(A1)*
   624000 (m^3) *(AG)*

> **Note:**   Both an unrounded answer that rounds to the given answer and the rounded value must be seen for the *(A1)* to be awarded.

                                                                        [3 marks]

Question 5 continued

(c) $(d =)$ -1800 **(A1)**

                                                                                        [1 mark]

(d) $(u_{13} =)$ $45000 + (13-1)(-1800)$ **(M1)**

Note:	Award **(M1)** for correct substitutions in arithmetic sequence formula. **OR** Award **(M1)** for a correct 4th term seen as part of list.

       23400 (m^3) **(A1)(ft)(G2)**

Note:	Follow through from part (c) for their value of d.

                                                                                        [2 marks]

(e)   (i) $0 = 45000 + (n-1)(-1800)$ **(M1)**

Note:	Award **(M1)** for their correct substitution into arithmetic sequence formula, equated to zero.

              $(n =)$ 26 **(A1)(ft)(G2)**

Note:	Follow through from part (c). Award at most **(M1)(A0)** if their n is not a positive integer.

       (ii) 25 **(A1)(ft)**

Note:	Follow through from part (e)(i), but only if their final answer in (e)(i) is positive. If their n in part (e)(i) is not an integer, award **(A1)(ft)** for the nearest lower integer.

                                                                                        [3 marks]

(f) $(S_8 =)$ $\dfrac{8}{2}\big(2 \times 45000 + (8-1) \times (-1800)\big)$ **(M1)**

Note:	Award **(M1)** for their correct substitutions in arithmetic series formula. If a list method is used, award **(M1)** for the addition of their 8 correct terms.

       310000 (m^3) $(309\,600)$ **(A1)(ft)(G2)**

Note:	Follow through from part (c). Award at most **(M1)(A0)** if their final answer is greater than $624\,000$.

                                                                                        [2 marks]

Question 5 continued

(g) $(S_{25} =)$ $\dfrac{25}{2}\big(2 \times 45000 + (25-1) \times (-1800)\big)$, $(S_{25} =)$ $\dfrac{25}{2}\big(45000 + 1800\big)$ **(M1)**

> **Note:**   Award **(M1)** for their correct substitutions into arithmetic series formula.

$S_{25} = 585000$ (m^3) **(A1)(ft)(G1)**

> **Note:**   Award **(M1)(A1)** for correctly finding $S_{26} = 585000$ (m^3), provided
>
> working is shown e.g. $(S_{26} =)$ $\dfrac{26}{2}\big(2 \times 45000 + (26-1) \times (-1800)\big)$,
>
> $(S_{26} =)$ $\dfrac{26}{2}\big(45000 + 0\big)$. Follow through from part (c) and either their
>
> (e)(i) or (e)(ii). If $d < 0$ and their final answer is greater than $624\,000$,
> award at most **(M1)(A1)(ft)(R0)**. If $d > 0$, there is no maximum, award
> at most **(M1)(A0)(R0)**. Award no marks if their number of terms is not
> a positive integer.

585000 $(\text{m}^3) < 624000$ (m^3) **(R1)**
Hence it will never be filled   **(AG)**

> **Note:**   The **(AG)** line must be seen. If it is omitted do not award the final **(R1)**.
> Do not follow through within the part.
> For unsupported $\big(S_{25} =\big)$ 585000 seen, award at most **(G1)(R1)(AG)**.
> Working must be seen to follow through from parts (c) and (e)(i) or (e)(ii).

OR

$S_n = \dfrac{n}{2}\big(2 \times 45000 + (n-1) \times (-1800)\big)$ **(M1)**

> **Note:**   Award **(M1)** for their correct substitution into arithmetic series formula, with n.

Maximum of this function 585225 (m^3) **(A1)**

> **Note:**   Follow through from part (c). Award at most **(M1)(A1)(ft)(R0)** if their final answer
> is greater than $624\,000$. Award at most **(M1)(A0)(R0)** if their common difference
> is not -1800. Award at most **(M1)(A0)(R0)** if 585225 is not explicitly identified as
> the maximum of the function.

585225 $(\text{m}^3) < 624000$ (m^3) **(R1)**
hence it will never be filled   **(AG)**

> **Note:**   The **(AG)** line must be seen. If it is omitted do not award the final **(R1)**.
> Do not follow through within the part.

continued…

Question 5(g) continued

OR

sketch with concave down curve **and** labelled 624000 horizontal line   ***(M1)***

> **Note:** Accept a label of "tank volume" instead of a numerical value. Award
> ***(M0)*** if the line and the curve intersect.

curve explicitly labelled as

$$S_n = \frac{n}{2}\big(2 \times 45000 + (n-1) \times (-1800)\big) \text{ or equivalent} \qquad \textbf{\textit{(A1)}}$$

> **Note:** Award ***(A1)*** for a written explanation interpreting the sketch. Accept a
> comparison of values, e.g $585225 \,(\mathrm{m}^3) < 624000 \,(\mathrm{m}^3)$, where 585225
> is the graphical maximum. Award at most ***(M1)(A0)(R0)*** if their common
> difference is not -1800.

the line and the curve do not intersect   ***(R1)***
hence it will never be filled   ***(AG)***

> **Note:** The ***(AG)*** line must be seen. If it is omitted do not award the final ***(R1)***.
> Do not follow through within the part.

OR

$$624000 = \frac{n}{2}\big(2 \times 45000 + (n-1) \times (-1800)\big) \qquad \textbf{\textit{(M1)}}$$

> **Note:** Award ***(M1)*** for their correctly substituted arithmetic series formula
> equated to 624000 (623538).

Demonstrates there is no solution   ***(A1)***

> **Note:** Award ***(A1)*** for a correct working that the discriminant is less than zero **OR**
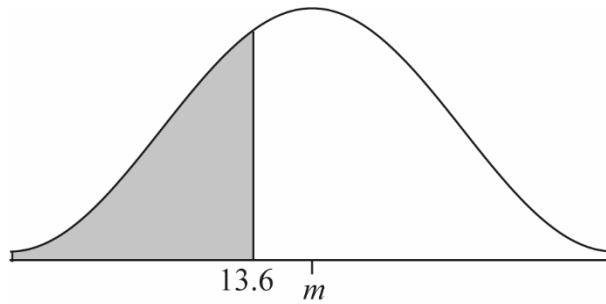> correct working indicating there is no real solution in the quadratic formula.

There is no (real) solution (to this equation)   ***(R1)***
hence it will never be filled   ***(AG)***

> **Note:** At most ***(M1)(A0)(R0)*** for their correctly substituted arithmetic series formula
> $= 624000$, 623538 or 622800 with a statement "no solution". Follow through
> from their part (b).

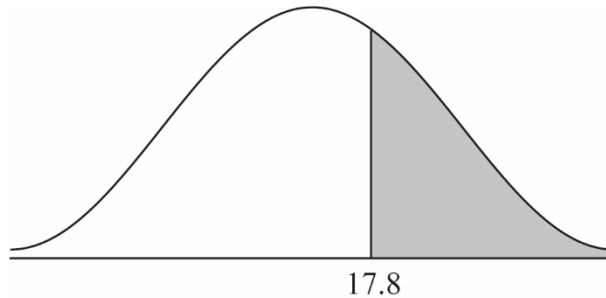***[3 marks]***

Total [16 marks]

6.   (a)

(A1)(A1)

Note:	Award **(A1)** for bell shaped curve with mean m **or** 13.6 indicated. Award **(A1)** for approximately correct shaded region.

[2 marks]

(b) $P(T > 17.8) = 0.3$ **(M1)**

   OR

(M1)

Note:	Award **(M1)** for correct probability equation using 0.3 **OR** correctly shaded diagram indicating 17.8. Strict or weak inequalities are accepted in parts (b), (c) and (d).

$$\frac{13.6+17.8}{2} \quad \left(17.8 - \frac{17.8-13.6}{2}\right) \textbf{ OR } \left(13.6 + \frac{17.8-13.6}{2}\right)$$ **(M1)**

Note:	Award **(M0)(M1)** for unsupported $\dfrac{13.6+17.8}{2}$ **OR** $\left(17.8 - \dfrac{17.8-13.6}{2}\right)$
	OR $\left(13.6 + \dfrac{17.8-13.6}{2}\right)$ **OR** the midpoint of 13.6 and 17.8 is 15.7.
	Award at most **(M1)(M0)** if the final answer is not seen. Award **(M0)(M0)** for using known values $m = 15.7$ and $\sigma = 4$ to validate $P(T < 17.8) = 0.7$ or $P(T < 13.6) = 0.3$.

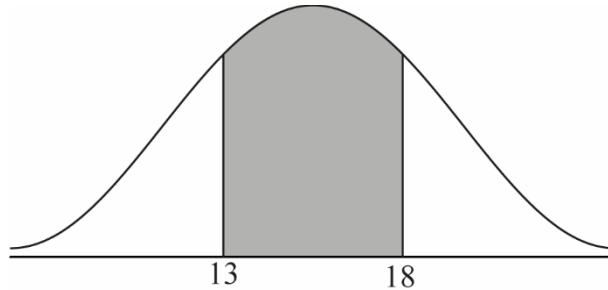   15.7   **(AG)**

[2 marks]

Question 6 continued

(c)   (i) $P(13 \leq T \leq 18)$ *(M1)*

        OR

                                                                          (M1)

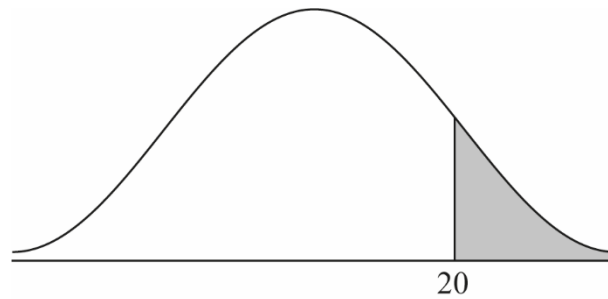| **Note:**	Award *(M1)* for correct probability equation **OR** correctly shaded diagram indicating 13 and 18.

             0.468 (46.8%, 0.467516…)   *(A1)(G2)*

    (ii) $P(T \geq 20)$ *(M1)*

        OR

                                                                          (M1)

Note:	Award *(M1)* for correct probability equation **OR** correctly shaded diagram indicating 20.

             0.141 (14.1%, 0.141187…)   *(A1)(G2)*
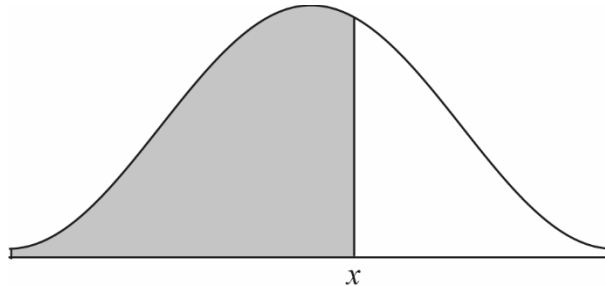
                                                                          [4 marks]

Question 6 continued

(d) $P(T < t) = 0.6$ **(M1)**

    OR

x

                                                                     (M1)

> **Note:**   Award **(M1)** for correct probability equation **OR** for a correctly shaded region with x indicated to the right-hand side of the mean.

    16.7   (16.7133...)   **(A1)(G2)**

                                                                     [2 marks]

(e) $0.467516\ldots \times 0.141187\ldots \times 2$ **(M1)(M1)**

    OR

    $(0.467516\ldots \times 0.141187\ldots) + (0.141187\ldots \times 0.467516\ldots)$ **(M1)(M1)**

> **Note:**   Award **(M1)** for the multiplication of their parts (c)(i) and (c)(ii), **(M1)** for multiplying their product by 2 or for adding their products twice. Follow through from part (c).

    0.132   (13.2 %, 0.132014...)   **(A1)(ft)(G2)**

> **Note:**   Award **(G0)** for an unsupported final answer of 0.066007...

                                                                       [3 marks]

(f) $\dfrac{69}{102} \times 200$ **(M1)**

> **Note:**   Award **(M1)** for correct probability multiplied by 200.

    135 (135.294…)   **(A1)(G2)**

                                                                       [2 marks]

Question 6 continued

(g)　$\left(\dfrac{67}{98} \times 200 =\right)$　136.734...　　　　　　　　　　　　　　　　*(A1)*

> **Note:** Award *(A1)* for 137 or 136.734… seen.

　　Emlyn is incorrect, $135 < 137$　$(135.294... < 136.734...)$　　　　*(R1)*

> **Note:** To award the final *(R1)*, both the conclusion and the comparison must be seen.
> Award at most *(A0)(R1)***(ft)** for consistent incorrect methods in parts (f) and (g).

　　OR

$\left(\dfrac{67}{98} =\right)$　0.684　(0.683673...)　$\left(\dfrac{69}{102} =\right)$　0.676　(0.676470...)　　　*(A1)*

> **Note:** Award *(A1)* for both correct probabilities seen.

　　Emlyn is incorrect, $0.676 < 0.684$　　　　　　　　　　　　　　*(R1)*

> **Note:** To award the final *(R1)*, both the conclusion and the comparison must be seen.
> Award at most *(A0)(R1)***(ft)** for consistent incorrect methods in parts (f) and (g).

[2 marks]
Total [17 marks]